Dynamic analysis of a double Hopf 1:2 resonance in a delay differential equation via a frequency-domain method

It is considered a second order differential equation with one delay and a quadratic nonlinearity, which includes three additional parameters. This model exhibits two equilibrium points, whose stability was analyzed completely. Besides, some particular parameter configurations were found where some different resonant double Hopf bifurcations take place, in particular of type 1:2. It is known that in a neighborhood of this singularity, limit cycles with frequency ω or 2ω appear singly but also simultaneously. Moreover, the existence of period doubling bifurcations of cycles is frequent in the described context. Related with harmonic balance methods and dynamic systems control, the frequency domain methodology allows, via the graphical Hopf bifurcation theorem, the detection of Hopf bifurcations and the attainment of approximate expressions for the rising of periodic solutions. Thus, different dynamic features were analyzed in the unfolding of this singularity like: the number of existing limit cycles associated to one or other frequency as well as the cycles stability over the Hopf bifurcations curves. Also, saddle-node, period-doubling and torus (or Neimark-Sacker) bifurcations of cycles were detected and their associated curves were obtained in some parameter plane. These results were established starting from fourth order harmonic balance approximations of the periodic solutions, coming through the selected methodology. Then, one Tchebyschev collocation method is applied to build a finite approximation of the monodromy operator and finally the relevants Floquet multipliers were computed. All the achieved results were checked with those coming from well-known softwares for delay differential equations, showing the local effectiveness of the used method.