Goodness-of-fit Based Weather Radar Ground Clutter Model Selection

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***Abstract*—In many studies, different models for weather radar clutter signal are used, each with its advantages and disadvantages. In this paper we use a model selection method through a goodness-of-fit (GoF) test over its power spectral density. The Barlett's T_p test's performance is firstly studied using synthetic radar data. This specific test has the advantage of being model independent. Using this test we compare the GoF of different clutter models to real measurement data obtained from an Argentinian weather radar (Radar Meteorológico Argentino, RMA). The Gaussian shape for the Power Spectral Density (PSD), both with and without considering windowing effects, and a first order autoregressive (AR) model are evaluated, since they are the most popular in weather radar applications. We also suggest truncating the spectrum to the clutter mode because it shows an improvement for the model selection. As a result, the first order AR model offers a higher rate of test acceptance than the other models.**

***Index Terms*—Goodness-of-fit, Power Spectral Density, Clutter, Doppler Weather Radar**

I. Introduction

One of the most challenging tasks in weather radar signal processing is getting rid of the unwanted reflections due to enviromental agents such as ground, trees, mountains, etc. This signal component, referred to as *clutter*, is usually present and its high power level can make the meteorological target component imperceptible. To decide whether clutter is present or not, it is important to know its properties and to build a model to describe it. Many studies have already been carried out to model weather signal spectra. Such models are commonly related to the Power Spectral Densities (PSD) shape and its spectral moments. Janssen [1] derived the Gaussian PSD model from theoretical arguments and showed that it fits correctly -in a least squared error sense- to measured data in more than 75% of the cases.

Pinsky et. al. [2] proposed the first order autoregressive (AR) model for the PSD and derived an iterative algorithm to estimate the model parameters. A complete description and analysis of ARMA models for radar signals has been presented by Thomas & Haykin [3].

It is worth mentioning that each model implies a particular group of parameters and also different estimation criteria. A variety of estimators for both frequency and time domain have been proposed [2], [4]–[7].

Hypothesis tests have been generally evaluated over the probability distribution function (PDF). For example, Billingsley [8] compares Rayleigh, Weibull, Log-Normal, and K-distribution models for ground clutter data using GoF tests. Additionally, Zhu Ling [9] explains how Gaussian models are not the best fitting models by making Kolmogorov-Smirnov tests over real data. Nevertheless, none of the aforementioned studies show how tight the PSD adjusts to the measurements.

Although there are practical solutions that mitigate the clutter without requiring an exact model for the PSD as in [10], [11] or assuming a Gaussian model as in [12], we hope that the use of more precise models will lead to implementations with better detection and mitigation performance.

In this paper we propose measuring how well the first order AR process and the Gaussian PSD shape model the ground clutter for weather radar applications. We based our work on a goodness-of-fit (GoF) analysis by directly testing the PSD shape for these models. A particular model selection test was chosen due to its model independence behaviour in a statistical sense. The goodness of each model will be measured in terms of acceptance rate over measured data cells.

The paper is organized as follows. The signal models are described in Section II. We give the expressions for the PSD in each case and we define their parameters. In Section III we introduce the Barlett's T_p test used to measure the GoF of the model to clutter data. Using real weather radar data we evaluate the test and present the results in Section IV. Finally, Section V summarizes the work and presents conclusions.

II. Radar signal modelling

There are several models suitable for weather radar signal PSD. Among them, the Gaussian is the most accepted. Its properties are well known and its PSD expression depends explicitly on the spectral moments.

On the other hand, the autoregressive models are popular due to their versatility. With a few number of parameters the AR process can model different spectrums. The first order model is particularly used in many applications.

Each model is not only characterized by its PSD but also by the available estimation criteria for its parameters. This is an important issue, because the data sets are relatively small, thus a windowed version of the signal should be considered. Taking into account the effect of the window in the model its

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complexity increases both in its definition and its estimation methods. This consideration is usually avoided for simplicity.

We are interested in modeling the clutter in the weather radar signal using the two mentioned models for the PSD signal and a third model including the window effect for the Gaussian PSD.

- **Gaussian model**: the PSD is given by

$$S_G(f) = \frac{S_c}{\sqrt{2\pi}\sigma_c} e^{-\frac{(f-f_c)^2}{2\sigma_c^2}} + \frac{N_0}{2}, \quad (1)$$

where S_c, f_c and σ_c are respectively the power, the mean frequency –expected to be approximately zero– and spectrum width of the clutter, and $N_0/2$ is the noise power spectral density. In short, we define the parameter vector as

$$\theta_G = [S_c \; f_c \; \sigma_c \; N_0/2]^T. \quad (2)$$

- **First order AR model**: the PSD is given by

$$S_{AR}(f) = \frac{\sigma_\epsilon^2}{\left|1 - a_1 e^{j(2\pi f/f_s)}\right|^2} + \frac{N_0}{2}, \quad (3)$$

where a_1 is the AR(1) process coefficient, σ_ϵ^2 is the innovation variance and and $N_0/2$ is the noise level. The parameter vector is

$$\theta_{AR} = [a_1 \; \sigma_\epsilon^2 \; N_0/2]^T. \quad (4)$$

- **Windowed Gaussian model**: the PSD is a modified version of the Gaussian model PSD, given by

$$S_{G_W}(f) = \{S_a * W_2\}(f) + \frac{N_0}{2}, \quad (5)$$

where $S_a(f)$ is the asymptotic clutter PSD

$$S_a(f) = \frac{S_c}{\sqrt{2\pi}\sigma_c} e^{-\frac{(f-f_c)^2}{2\sigma_c^2}}, \quad (6)$$

and $W_2(f) = |W(f)|^2$, being $W(f)$ the Discrete Time Fourier Transform (DTFT) of the rectangular window. It should be remarked that the length of the window is always N. The parameter vector is the same as in (2).

If we denote the complex set of samples at the quadrature demodulator's output as $\boldsymbol{y} = [y_1 \; \cdots \; y_N]^T$, the hypotheses test can be formulated as

$$\begin{cases} H_0 : S_{yy}(f) = S_0(f) \\ H_1 : S_{yy}(f) \neq S_0(f), \end{cases} \quad (7)$$

where the decision between H_0 and H_1 is taken after $\boldsymbol{y}$ has been received.

Under the null hypothesis H_0, it is assumed that the PSD of the clutter data matches with the PSD model, $S_0(f)$. Under the hypothesis H_1 it is assumed that the measurement has a PSD different to $S_0(f)$.

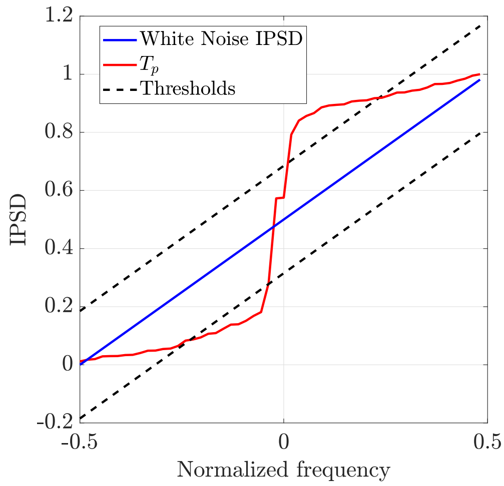

Fig. 1. Signal IPSD, White Noise PSD and thresholds for the Barlett's T_p test.

III. Barlett's T_p test

To test the PSD goodness-of-fit we use the Barlett's T_p test [13]. It is derived as an adaptation of the Kolmogorov-Smirnov test, using the *whitened* integrated power spectral density (IPSD) instead of the cumulative probability function. This test has the advantage of possessing a PSD independent statistic. The chosen PSD function will not affect the statistic's distribution, but its number of samples will. The test's statistic is defined as

$$T_p = \frac{\sum_{n=1}^{p} \hat{S}_{yy}[n]/S_0(f_n)}{\sum_{n=1}^{N} \hat{S}_{yy}[n]/S_0(f_n)}, \quad (8)$$

where $\hat{S}_{yy}[n]$ is the PSD estimator known as *Welch's periodogram* [14] and $f_n = \frac{f_s}{N}(n-1) - \frac{f_s}{2}$, with $n = 1, \ldots, N$, are the ordinate's frequencies. The test procedure consists in deciding whether T_p corresponds to a *white* noise process or not, by comparing their IPSD. Thus, the null hypothesis is rejected if

$$D_N = \max_p \{|T_p - p/N|\} \geq a/\sqrt{N}, \quad (9)$$

where $a/\sqrt{N}$ is a threshold fixed by the significance level α of the test. Thereof, from the D_N's probability distribution it can be shown that

$$\lim_{N\to\infty} \mathrm{P}\{D_N \leq a|H_0\} = 1 - \alpha, \quad (10)$$

with

$$\alpha = 1 - \sum_{j=-\infty}^{\infty} (-1)^j e^{-2a^2j^2}. \quad (11)$$

Fig. 1 shows an example of a IPSD from a clutter echo signal where the thresholds are crossed and the test fails. When analyzing a set of cells corresponding to different coherence intervals, it is important to recall that the theoretical PSD will

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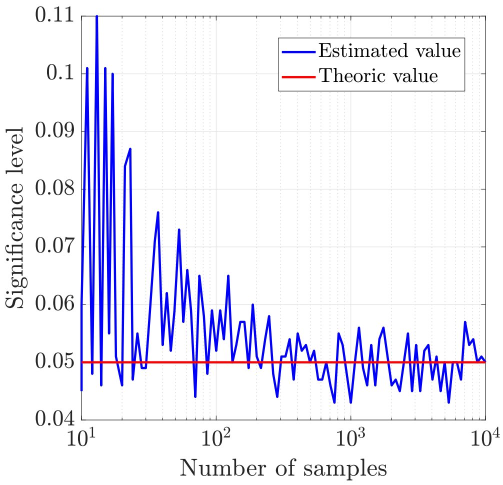

Fig. 2. Significance level estimation versus number of samples used in periodogram. Theoretical value of $\alpha = 0.05$.

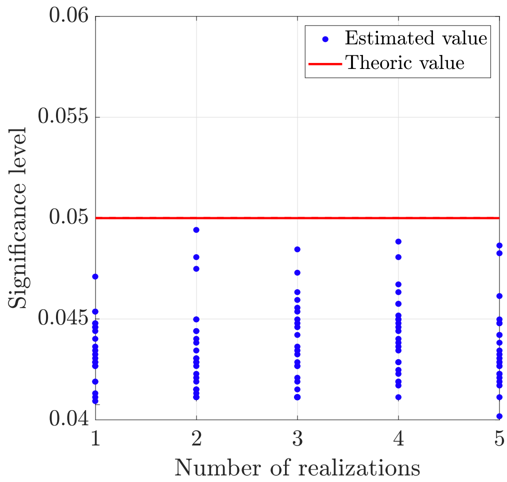

Fig. 3. Significance level estimation versus number of realizations used in periodogram. Theoretical value of $\alpha = 0.05$.

be different for each cell. Therefore each null hypothesis will be different, i.e,

$$S_0(f_n)_{r,\phi} \rightarrow S_0(f_n|\hat{\theta}_{r,\phi}), \tag{12}$$

where $\hat{\theta}_{r,\phi}$ is the parameter estimation vector of each cell. If the amount of cells is large enough, the model selection can be considered accurate if the percentage of accepted tests matches the value $1 - \alpha$.

A. Test's performance

Before applying the tests over the samples it is important to know its performance and limitations. In order to do that, two simulations were performed over synthetic data with controlled statistics. The first test consisted in studying the Type I Error Probability dependence with the number of samples used for the PSD estimation. A significance level of $\alpha = 0.05$ was fixed. Fig. 2 shows the results of the test using Gaussian PSD synthetic data. As the number of samples increases the estimated error probability converges to the expected theoretical value, according to (10).

On the second test, the number of samples was fixed, changing the number of realizations I used for the periodogram. It is worth mentioning that this change involves modifying the T_p's definition, since the *Welch periodogram* strictly uses one realization. However, it was noted that D_N's distribution did not change significantly. Fig. 3 shows the estimated significance level versus I. Results indicate that there is no benefit in using more than one realization.

B. Spectrum truncation

Since our study focuses on spectrums were clutter predominates a typical PSD shape is assumed, as shown in Fig. 4, where the PSD was estimated using the Welch periodogram over the measurments described in Section IV. Observing Fig. 4 the spectrum can be divided into two regions: the central region where the clutter mode is present with a high power level and border regions where noise is predominant and power

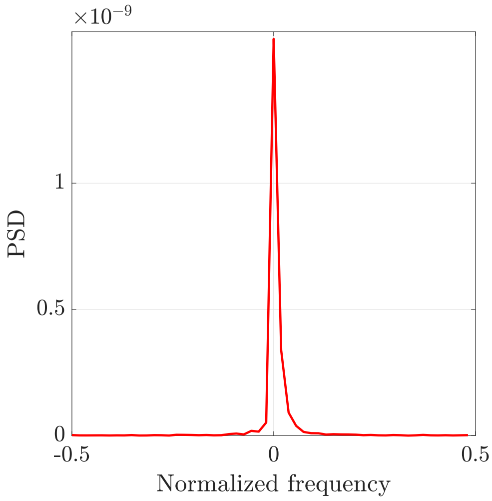

Fig. 4. Typical estimated PSD shape of clutter and noise radar echoes.

levels are low. It can also be seen that clutter extends over a narrow band. Since clutter to noise ratios (CNR) are usually between 10 to 50 dB, the noise region entails a problem when normalizing the spectrum due to its low sample values, but large total power value. This implies that the IPSD will not *grow* as fast as it should, crossing the threshold as seen in the example of Fig. 1. Hence, truncating the spectrum to the clutter region is advantageous. Reducing the number of samples in this case does not imply increasing the significance level, as shown in Fig. 2, since the truncation is done in the frequency domain and the amount of samples per frequency interval remains constant.

IV. Results

Below, results of the goodness-of-fit of mentioned models to real clutter data are presented. Measurements of the RMA-1 Argentinian weather radar, located in Córdoba city were used. The RMA-1 is C-band polarimetric radar, designed and developed by the company INVAP. Specifically, the used data

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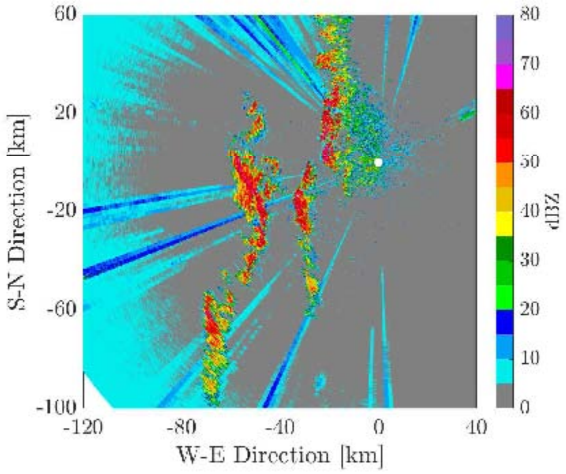


Fig. 5. Reflectivity factor estimated in Córdoba city's surroundings.

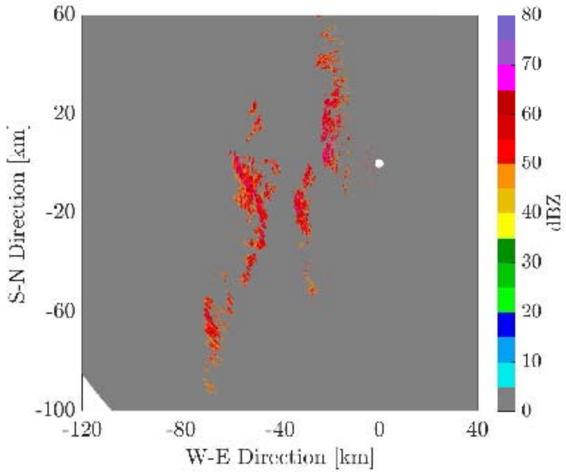
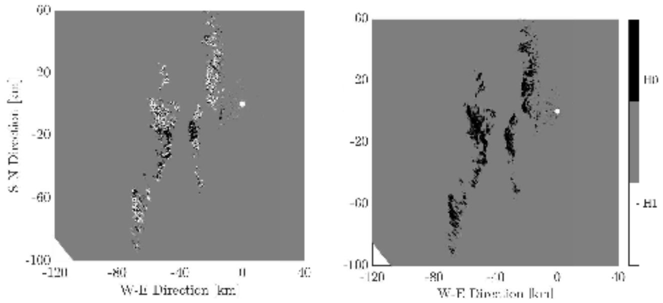


Fig. 6. Cell selected as clutter only ($Z > 45$ dBz).

was recorded on September 7, 2018, at 2:28 pm (UTC-03:00). The shown results correspond to a complete sweep of the horizontal polarization (HH) at the elevation angle 0.5 degrees under clear sky conditions, ensuring that there were no weather components present.

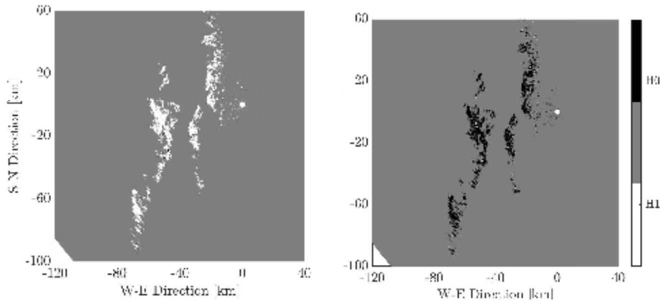
Fig. 5 shows the reflectivity factor, Z, of the measured data. A region of high reflectivity due to ground clutter can be seen, which can be explained by the presence of hills in the surrounding environment. Moderate reflectivity lines can also be seen at a few azimuth angles, corresponding to electromagnetic interference probably due to WLAN networks. Since we are interested in modeling clutter signals, only cells with reflectivity greater than 45 dBz were taken into account. These cells are shown in Fig. 6.

In Figs. 7-9, the untested cells are represented in grey, the cells where the test was accepted (i.e. H_0 has not been rejected) are represented in black, and the cells where the test failed (i.e. H_0 has been rejected) are represented in white. To show the effect of truncating the spectrum, the test was done on three sets of samples: $N = 54$ (complete spectra), and



(a) (b)

Fig. 7. Hypothesis test results for (a) complete spectra and (b) 8-sample-truncated spectra assuming the first order autoregressive model.



(a) (b)

Fig. 8. Hypothesis test results for (a) complete spectra and (b) 8-sample-truncated spectra assuming Gaussian model

$N = 8$ and 16 centered samples (central mode). In all of the cases the test's significance level was set to $\alpha = 0.05$.

For all models used, it is necesary to determine the noise level $N_0/2$. This was performed in the spectrum domain following the approach presented in [15].

A. First order autoregressive model

In Fig. 7 the results for the AR(1) model are shown. The criterion used for estimating the AR(1) process parameters is maximum asymptotic likelihood [16]. In the case where $N = 54$ (Fig. 7.a), only 17% of the tested cells had H_0 accepted. Note that at angles where the interference was present, the rejection rate is higher. On the other hand, in the case where $N = 8$ (Fig. 7.b) the 100% of the tested cells were accepted, having a lesser Type I error probability than expected.

B. Gaussian model

Figs. 8.a-8.b show the test results assuming the Gaussian PSD model. Its parameters where estimated using the Riemann approximation of the integrals that define the spectral moments. Complete PSD tests, $N = 54$, only had 1% success rate, while truncated PSD tests, $N = 8$, where accepted 93% of the times. It is wise to remark that Riemann estimation algorithm was done with the complete spectra in both cases.

C. Windowed Gaussian model

The last study was made applying the windowed-Gaussian model. Results are shown in Figs. 9.a-9.b for complete and

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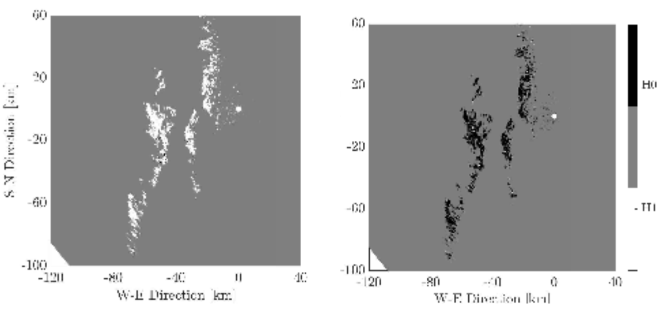

(a) (b)

Fig. 9. Hypothesis test results for (a) complete spectra and (b) 8-sample-truncated spectra assuming Gaussian model with window correction.

truncated, $N = 8$ samples, spectra relatively. In the first case, 1% of tests where accepted whereas 95% in the second case. The slight improvement regarding the previous model makes sense with the truncation since the window effect is stronger in the central mode.

The 16-sample-truncated spectra results are shown in table I. As seen, truncated spectra improved the test results for every model. But the window correction did not make any significant change.

Model \ Samples	54 (all)	16	8
AR(1)	17%	96%	100%
Gaussian	1%	57%	93%
Windowed G	1%	57%	95%

TABLE I
PERCENTAGE OF CLUTTER CELLS WITH ACCEPTED H_0 FOR DIFFERENT MODELS AND SPECTRUM TRUNCATIONS.

V. Conclusion and discussion

In this study, a PSD goodness-of-fit test has been presented as a model selection criterion for ground clutter data in weather radar applications.

The three most popular models used in weather radar applications were described and evaluated. These are the Gaussian model, both with and without considering windowing effects, and the first order autoregressive model.

The Barlett's T_p test, an adaptation of the Kolmogorov-Smirnov test for the PSD, was studied and used for testing the proposed models. Firstly, its performance and limitations were analyzed by means of synthetic weather radar data, showing that large number of samples improves the tests performance. Secondly, we applied this test to compare the GoF of different clutter models to real measurement data obtained from the Radar Meteorológico Argentino 1 (RMA-1).

In the particular case of ground clutter, it was noted that only central mode spectrum samples are useful. It was shown how noise spectra samples deteriorate the test's performance remarkably. A dynamic truncation of spectra as a function of clutter's width is believed to improve the test performance. Moreover, since the high power level clutter echoes is not the only parameter to detect clutter, using another way to select clutter cells is recommended.

Regarding the windowed Gaussian model, it is concluded that windowing considerations are not necessary. The reason why windowing did not make any improvement to the Gaussian model is due to the low frequency resolution commonly available in weather radar signals. Even though, changes in windowing parameters such as type and length should be studied.

Despite the Gaussian model being theoretically grounded, T_p tests showed that the AR(1) model fits better for clutter PSDs. Anyhow, the Gaussian model is not to be blamed independently since other estimation criteria for the parameters have not been evaluated.

A possible next step consists in evaluating the Barlett's T_p test for the Gaussian model using the pulse-pair-processing [17] algorithm to compute its parameters.

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