Asociación Argentina de Astronomía
BAAA, Vol. 55, 2012
A.C. Rovero, G.E. Romero, L. Pellizza & M. Lares, eds.

PRESENTACIÓN MURAL

Compact Stars in R-Squared Gravity

Federico García[1,2], Florencia A. Teppa Pannia[2], Mariana Orellana[1,2], Gustavo E. Romero[1,2]

(1) Instituto Argentino de Radioastronomía CCT La Plata - CONICET, C.C.5, (1894) Villa Elisa, Buenos Aires, Argentina.
(2) Facultad de Ciencias Astronómicas y Geofísicas, UNLP, Paseo del Bosque s/n, B1900FWA La Plata, Argentina.

Abstract. Recent progress in testing certain modified-gravity theories has been achieved by the study of the structure of compact objects like Black Holes and Neutron Stars (NSs). In particular, it is possible that the *squared*-gravity generalization of the Lagrangian density, given by the function $f(R) = R + \alpha R^2$, may yield acceptable models for heavier NSs than General Relativity (GR) does.

In the strong gravitational regime, the differences among alternative gravity theories should be enhanced. Because of the complexity of the field equations, perturbative methods are required to approach solutions. In this work, we calculate the structure of both NSs and Quark Stars (QSs) focusing on the behaviour of their interior profiles, compared to those from GR. We find that these profiles depend on high-order derivatives of the Equations of State (EoSs), showing regions where the enclosed mass decreases with the radius in a counter-intuitive way.

Resumen. En los últimos años se han registrado avances en la verificación de teorías de gravedad modificada a través del estudio de la estructura de objetos compactos. En particular, es posible que la generalización *cuadrática* de la densidad lagrangiana, dada por la función $f(R) = R + \alpha R^2$, permita modelar estrellas de neutrones más masivas que las admitidas por la Relatividad General (RG).

En el régimen de campo gravitacional fuerte, las diferencias entre teorías alternativas de la gravedad deberían amplificarse. Debido a la complejidad de las ecuaciones de campo, se requieren métodos perturbativos para aproximar las soluciones. En este trabajo calculamos la estructura de estrellas de neutrones y estrellas de *quarks* enfocándonos en el comportamiento de los perfiles interiores resultantes, y comparándolos con sus equivalentes en RG. Encontramos que estos perfiles dependen de las derivadas de order superior de la ecuación de estado utilizada, mostrando regiones donde la masa encerrada decrece con el radio, de modo contraintuitivo.

1. Introduction

The appearance of Extended Theories of Gravity was strongly stimulated by the possibilities they migth provide to reinterpret the current cosmological data (de Felice & Tsujikawa 2010; Sotiriou & Faraoni 2010) without involving non-standard matter in the energy-momentum tensor. We focus our work on the so-called $f(R)$-gravity theories, which are based on a modification of the Einstein-Hilbert action: the usual Lagrangian density is generalized replacing the Ricci curvature scalar, R, by a function of it or by a combination of high-order invariants of the curvature tensor (see, for instance, Capozziello & Faraoni 2011).

The simplest choice $f(R) = R + \alpha R^2$, also called "R-squared" gravity, had been further studied as a viable alternative cosmological model. But in contrast to gravity in the weak-field regime, gravity in the strong-field regime is largely unconstrained by observations (e.g. DeDeo & Psaltis 2003). Under this motivation, we investigate in detail the structure of NSs and QSs in R-squared gravity, using a perturbative approach (Babichev & Langlois 2010).

2. Structure equations in R-*squared* gravity

Modified TOV equations can be obtained from the gravitational field equations, assuming spherical symmetry and staticity. In the metric formalism, the variation of the action with respect to the metric yields fourth-order differential equations. We adopt the perturbative approach presented in Cooney et al. (2010) to solve them in the case that $f(R) = R + \alpha R^2 = (1 + \beta)R$, where the dimensionless quantity $\beta \equiv \alpha R$ comprises the deviation from GR and the perturbative method is applied as long as $|\beta| \ll 1$.

In this approach, functions in the metric are expanded into a leading term, denoted with subscript 0, plus a corrective one (subscript 1) that is of first order in β. Following Arapoğlu et al. (2011), the hydrodynamic quantities are also defined perturbatively: $\rho = \rho_0 + \beta\rho_1$ and $p = p_0 + \beta p_1$, and the mass is defined assuming that the solution for the metric has the same form as the exterior Schwarzschlid solution in GR. With this considerations, and taking into account that ρ_0 and p_0 satisfy Einstein's equations, the derived modified TOV equations are:

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho - 2\beta\left[4\pi r^2\rho_0 - \frac{c^2}{8G}r^2 R_0 + \left(2\pi\rho_0 r^3 - \frac{c^2}{G}r + \frac{3}{2}m_0\right)\frac{R_0'}{R_0} - r\left(\frac{c^2}{2G}r - m_0\right)\frac{R_0''}{R_0}\right]$$

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{c^2\rho + p}{2Gm - c^2 r}G\left\{\left(\frac{4\pi}{c^2}r^2 p + \frac{m}{r}\right) - 2\beta\left[\frac{4\pi}{c^2}r^2 p_0 + \frac{c^2}{8G}r^2 R_0 + \left(\frac{2\pi}{c^2}p_0 r^3 + \frac{c^2}{G}r - \frac{3}{2}m_0\right)\frac{R_0'}{R_0}\right]\right\}$$

where we use the prime for radial derivatives. Here the terms 2β [...] indicate the first order correction. Note that in order to work up to first order in β, the brackets have been evaluated at order zero. The α parameter is constrained to the values reported by Santos (2010), and references therein, which points to 10^8 cm$^2 < \alpha/3 < 10^{10}$ cm^2.

2.1. Equations of State and the numerical method

We calculate the mass-radius ($M_\star - R_\star$) relations for NSs, considering two different EoSs: SLY (Haensel & Potekhin 2004) and POLY (Silbar & Reddy

2004). The first one is a realistic EoS that represents the behaviour of nuclear matter at high density, but has a complex analytical representation and the error propagation of its derivatives is out of our knowledge. The second EoS is a simple polytropic approximation, $p \propto \rho^2$, and allows us to study zeroth-order modified gravity effects, separating them from more tricky issues arising from the analytical representation of the realistic EoS. The interior of QSs is represented by a SQM EoS, $p = (c^2\rho - 4B)/3$, assuming a typical value of $B = 60$ MeV fm^{-3} for the MIT bag constant (Glendenning 2000; Degrand et al. 1975).

3. Results and Conclusions

In Fig. 1 we show the mass-radius ($M_\star - R_\star$) relations using SLY, POLY and SQM EoSs, obtained for seven values of the α parameter between –0.2 and +0.2 km^2, and considering central densities, ρ_c, running from $10^{14.6}$ gr cm^{-3} to $10^{15.9}$ gr cm^{-3}. SLY EoS configurations are more sensitive to the value of α parameter than the others.

In the right-bottom panel of Fig. 1 we show the radial profiles obtained for the density, $\rho(r)$, and mass, $m(r)$, for $\alpha = -0.2$, 0.0 and +0.2 km^2 with the SLY EoS. Although the density, and hence the pressure, follow rather usual (resembling GR) profiles, the effects of $f(R)$ are reflected in the mass profiles and are evident close to the NS surface (i.e. at $r \sim 10$ km, for $\rho_c = 10^{15.4}$ gr cm^{-3}) where, in a narrow layer ($\Delta r \sim 0.2$ km), an unexpected (counter-intuitive) decrease in $m(r)$ appears before ($\alpha > 0$) or after ($\alpha < 0$) a dip (peak) in the profile. In the frame of GR, this effect could only be explained by means of a negative-density fluid, while in $f(R)$ theories, it arises as a natural consequence of the coupled space-time geometry and matter content. On the contrary, adopting a simple polytropic EoS for NSs and QSs, no anomalous behaviour of the internal profiles of the stars is observed. Hence, because these particular effects arrive as a direct consequence of the high-order derivatives of realistic EoSs, it is important to remark that the uncertainties on the analytical representation of the EoS adopted could have an enhanced effect on the $f(R)$ solutions.

Our most important result concerns the detailed study of the internal structure of NSs considering the largest acceptable values for the α parameter (i.e. the stronger perturbation allowed to GR by the actual constraints in R-squared gravity). We find that a combination between the metric behaviour and the high-order derivatives of the realistic EoS leads to a region where the mass enclosed in a spherical region of the star decreases with the radius in a counter-intuitive way.

Acknowledgments. The authors appreciate helpful comments from Prof. Santiago E. Pérez Bergliaffa. M.O. acknowledge support by the Argentine Agency CONICET and ANPCyT.

References

Arapoğlu S., Deliduman C., Yavuz Ekşi K., 2011, JCAP, 7, 20
Babichev E., Langlois D., 2010, Physical Review D, 81, 124051
Capozziello S., Faraoni V., 2011, Beyond Einstein Gravity. A survey of Gravitational Theories for Cosmology and Astrophysics. Springer Science+Business Media

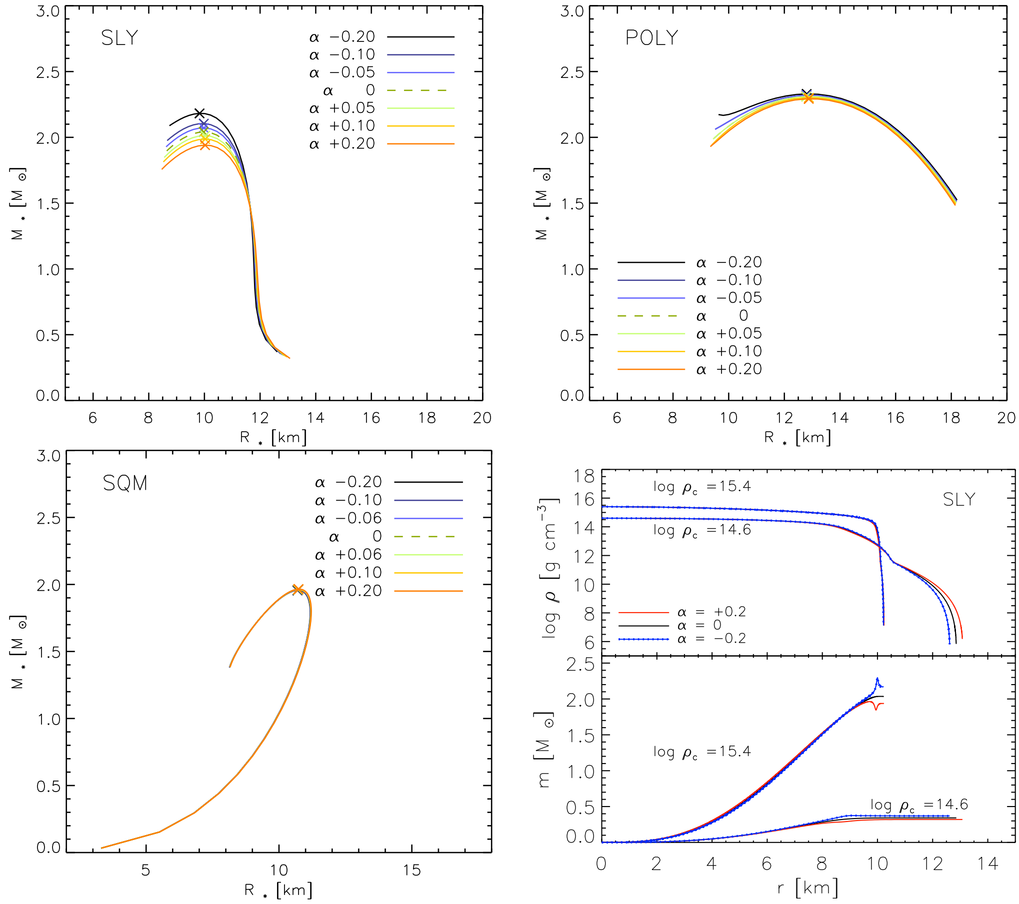

Figure 1. Mass-radius ($M_\star - R_\star$) relations for the three selected EoS. Crosses indicate the maximum mass for each curve, assuming a necessary condition for equilibrium: $\mathrm{d}M/\mathrm{d}\rho_c > 0$. Internal structure profiles of NSs with SLY EoS are shown in the bottom-right panel.

Cooney A., Dedeo S., Psaltis D., 2010, Physical Review D, 82, 064033
de Felice A., Tsujikawa S., 2010, Living Reviews in Relativity, 13, 3
DeDeo S., Psaltis D., 2003, Physical Review Letters, 90, 141101
Degrand T., et al., 1975, Phys.Rev.D, 12, 2060
Glendenning N. K., ed. 2000, Compact stars: nuclear physics, particle physics, and general relativity
Haensel P., Potekhin A. Y., 2004, A&A, 428, 191
Santos E., 2010, Physical Review D, 81, 064030
Silbar R. R., Reddy S., 2004, American Journal of Physics, 72, 892
Sotiriou T. P., Faraoni V., 2010, Reviews of Modern Physics, 82, 451