A DISJUNCTIVE MODEL TO ANALYZE AND REDEFINE THE LOGISTIC OF

REPLENISHING GOODS OF RETAILING STORES

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Abstract. This paper analyzes the distribution logistics for a retail business that has hundreds of stores located long distances from their warehouses. Given narrow revenue margins, retail companies must optimize the cost of the merchandise delivery to its stores. In general, the structure of the distribution consists of warehouses that concentrate goods which are then delivered to the stores according to a replenishment policy that contemplates its characteristics and location. The cost of the distribution logistics is significant and deserves special consideration. This paper presents a disjunctive multiperiod model to redesign the logistic infrastructure of delivering goods from warehouses to stores located in a wide geographical region. In the model, a warehouse can be installed, closed, expanded or replaced by a cross-docking terminal. The objective function is to minimize the cost of the whole distribution operation. A case study is presented to show the model capabilities.
Keywords: retail business – optimization- logistic structure – replenishment goods

1. INTRODUCTION

Distribution logistics is an important operation for retail business. Due to narrow revenue margins, companies must deliver goods on time with the lowest possible cost. For companies having hundreds of stores distributed nationwide, located long distances from their warehouses, the impact of the cost of delivering goods is significant. The configuration of the distribution net, delivery policies and the selection of optimal routes are important matters to overcome this problem (Christopher, 2016). The characteristics of the distribution logistics vary according to the type of the retail business, in general, the structure consists of warehouses that concentrate goods, which

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then are delivered to the sale stores following a replenishment pattern (Ballou, 2004). The replenishment is based on the store demand, which differs among locations (Agrawal and Smith, 2017). Trucks are the most used transportation medium because of its flexibility since they can go anywhere where there are roads. In some cases, a cross-docking terminal is used in the distribution structure in order to split the delivery into smaller parts to be sent to different locations or to reach places where large trucks are not permitted.

This work presents an optimization model to redesign the replenishment configuration of a retail company having sale stores located along a large geographical region. Despite the importance of retail business in the economy, papers in the literature are scarce in retail store operations and logistics. Mou et al. (2018) reviewed this situation, the authors analyzed about 255 papers from 32 scientific journals dealing with retail store operations, they discussed the limitations and opportunity areas to be studied. In particular, they established that in store logistics deserves more attention when multiple products are involved and the coordination between delivery and shelf replenishment is needed. Since those authors perform a review of scientific articles about retail store operations, it is an important reference to find papers related to that matter. Nevertheless, there are some works in the literature dealing with similar problems. Holzapfel et al. (2016) present a decision support model that integrates warehouse operations, transportation, and in store replenishment. They explore how the decisions on the delivery pattern affect retail operations and its logistics. A heuristic solution approach is developed to generate the same delivery pattern for each store. They provide a characterization of the retail network structure that was an inspiration source for this work. It served to define the distribution network among warehouses and the stores. The authors claim that having stable store clusters and tours, reduce short-term operational complexity and increase service quality. Cost improvements are on average 2.5 percent and up to 7 percent using simulated data and 1.5 percent in the case of a European grocery retailer.

Cardós and García-Sabater (2006) propose a model to design the retail chain inventory replenishment policy considering the transportation cost and inventory, the stock level and client service. They determine that in the design of a retail chain, the minimal cost to give a good service to the clients is a trade-off between the inventory level for each shop and delivery policies from central warehouses. For this study, the authors analyze the conjoint problem: a variation of the vehicle routing problem (VRP) as an extension of inventory management in the supply chain context. These authors analyze the delivery frequency and patterns together with the cost determination, which for our proposal becomes the definition of delivery routes, the travel distances, and the transportation cost. Milson and Smirnof (2016), apply a game theoretic framework to analyze the effects of transportation infrastructure where competing retail firms are located close to each other. The results show that companies which compete among them, locate their stores next to each other when they have access to important transportation infrastructure. The authors highlight the importance of analyzing the transportation infrastructure when considering the location of retail stores. These issues are relevant to decide the store location, the road infrastructure, and the role of having stores of the competence located nearby, but those aspects are not analyzed in this article.

Yao and Hsu (2009) propose a tree-based genetic algorithm (GA) to solve the configuration and the transportation planning of a multi-stage supply chain network. The objective is to determine the optimal locations of the distribution centers and optimal transportation routes to minimize the total costs of the whole system. The authors proposed the GA because they claim that a mixed integer nonlinear program formulation is not able to solve this problem efficiently because of its complexity.

Caridade et al. (2017) studied the warehouse activities and its costs in the logistic functions of the automotive industry. The goal is to restructure and optimize the efficiency of the warehouse operations. The study is performed by using the software Warehouse Management System (WMS) integrated into the company information system (IS). The methodology is, as a first step, to analyze the current situation to identify the aspects needing improvements then, to introduce new issues to the software, like eliminating a third-party warehouse that is rented, increase the capacity of an existing one, and reorganize the shelves. This new scenario is further analyzed. As a result, improvements are observed in the space management and its cost-efficiency, and also in the inventory administration. They tackle similar issues like the operation of existing warehouses, the installation of a new one and the activities inside them, but not the delivery logistics. Instead of a mathematical programming approach to find an optimum they used a specific software looking for just an improvement in the whole operation.

Rodriguez et al. (2014) present a strategic and tactical optimization model to redesign the supply chain of spare parts under demand uncertainty over a planning horizon divided in periods. The decisions involve new installations, expansions and elimination of warehouses and factories, inventory levels (safety stock and expected inventory) in distribution centers. The problem is formulated as a non-linear mixed -integer one; the authors use a piecewise linearization to obtain a tight lower bound of the optimal solution. The formulation is applied to the supply chain of electric motors but can be expanded to other industries. These authors propose a good formulation about the installation of new factories and warehouses, also the capacity expansion and elimination of existing ones, which was considered in our work. They contemplate investment and operation costs, but for transportation they do not include neither the distances traveled nor the number of trips, they use instead the demand and number of days.

Perona et al. (2001) studied the logistic chain management of the white goods industry on a sample of Italian companies, they analyzed the cost and the logistic chain, considering supply, production, distribution, and sales. The authors made an empirical analysis by collecting data from several companies, dealers, and customers, via interviews with people. Then they generated an empirical model to calculate the logistic chain and lost sale costs, considering different scenarios. They extracted three main conclusions, first they pointed out the relevance of the logistic cost and lost sales, which is about 30% of the overall cost, in a business where profitability is around 5%. Second, the distribution phase is responsible for about 80% of those costs, requiring a re-engineering of the chain. Third, they conclude that a better integration among the actors to exchange information would diminish the cost of lost sales. Although these authors studied a similar supply chain as our example (a white goods industry) they employed an empirical methodology as an alternative to mathematical programming, looking for improving the performance instead of finding the optimum. They highlight the importance of studying the logistic cost in the operation of a retail chain.

In a recent article, Tarapataa et al. (2020) present some components of a Distribution Optimization System based on data driven machine learning methods. The components of the proposal are based on algorithms of geocoding information to calculate the delivery address, route and regional distribution optimization and also a smart tracking device monitoring vehicle traffic parameters. The objective is the cost optimization of resources, transportation, delivery time and cost. At the time of the article the approach was in its initial stages. In the last ten years, data driven and machine learning are the “new era” methods which are successfully used in many industrial applications, and that was the reason to select this article. The authors proposed a method, but it is still a project, it is not supported with results, it needs more work to consider it an alternative to mathematical programming.

The model proposed in this article corresponds to a disjunctive multi period model to optimize the structure of the replenishment logistics of a retail company having stores distributed nationwide. Disjunctive programming (DP) provides a systematic modeling framework for problems involving discrete decisions (Trespalacios and Grossmann, 2012), it is an alternative approach for algebraic MILP/MINLP models. In the last two decades several problems can be found, formulated and solved using DP mainly in the area of Process Systems Engineering and Industrial Engineering. Some of those works published in the last five years are: Pedrozo et al. (2021) optimizes the design of an ethylene and propylene production plant, Wu et al. proposed a rolling horizon approach for scheduling of multiproduct batch plants (2021), Novas et al. (2020) solved the truck loading problem for a non-alcoholic beverage industry, Rodriguez et al. (2017) applied DP to a polyurethane foam plant, Castro et al (2021) present a disjunctive model to handling preemption in discrete and continuous-time formulations of scheduling problems. There are also works proposing algorithms, methods, techniques, and systems for DP. Ignacio Grossmann (2002) presents a review of nonlinear mixed-integer and disjunctive programming techniques where an overview of methods and techniques available at that time are raised. More recently, Sawaya and Grossmann (2012) and Ruiz and Grossmann (2012) present systematic basic steps for linear and nonlinear disjunctions, respectively, to sequentially transform them into a tighter relaxation of the disjunctive set to solve problems in fewer iterations, Trespalacios and Grossmann (2016) propose an algorithm based on cutting planes for the logic-based outer-approximation (LBOA) method to solve process synthesis problems. Regarding the codes, LOGMIP (Vecchietti and Grossman, 1999) was the first system to solve linear and non-linear disjunctive problems, Pyomo.GDP (Chen et al, 2018) is another alternative to formulate and solve a discrete model formulated via disjunctions. Nevertheless, even with the active research in disjunctive programming, no disjunctive models can be found for the solution of the replenishment logistic of a retail business, our model corresponds to a deterministic one, while those found in the literature use heuristic approaches, or they are solved by means of a problem-specific software. To emphasize the capabilities of our proposal, section 3 describes disjunctions to make discrete decisions related to changes in the facilities structure of the distribution network, including the cost involved in and the delivery routes employed to deliver merchandise. The objective function is the cost minimization of the whole distribution operation. After presenting the model formulation in section 3, an example is solved to show the model capabilities.

2. PROBLEM DESCRIPTION

The problem analyzed in this paper corresponds to merchandise replenishment to stores located around a wide geographical region over a time horizon divided in periods. Warehouses (WH) receive and concentrate the products that are delivered to the stores according to their needs. The replenishment structure can use cross dockings (XD) to deliver goods.

The structure of the replenishment can change over time, new WH and XD can be installed at specific locations, existing WH can continue its operation, even increasing its capacity, or can be closed. Existing cross dockings can continue their operation or not.

Several assumptions are made in the problem to solve it:

- The replenishment volume of products in m^3 to be delivered to a store is known.
- Store sales increase over the time horizon in an annual percentage.
- Trucks of different sizes are used to deliver merchandise.
- Transportation cost per truck size, from warehouses to stores is calculated based on the distances.
- Each store must be visited a minimum number of times over a period *(minv)*.
- Warehouse costs include labor, operation and stock.
- Seasonal overstock of products that exceeds warehouse capacity can happen and, for that reason, third-party warehouse spaces are rented for a short period of time.
- Trucks must be filled in a minimum percentage to transport products.
- Several delivery routes are defined from storage to the stores, the model selects one according to the amount to be delivered, the truck size, the stores to be visited and the stores located in the routes. In order to understand this assumption, we are presenting Fig.1.

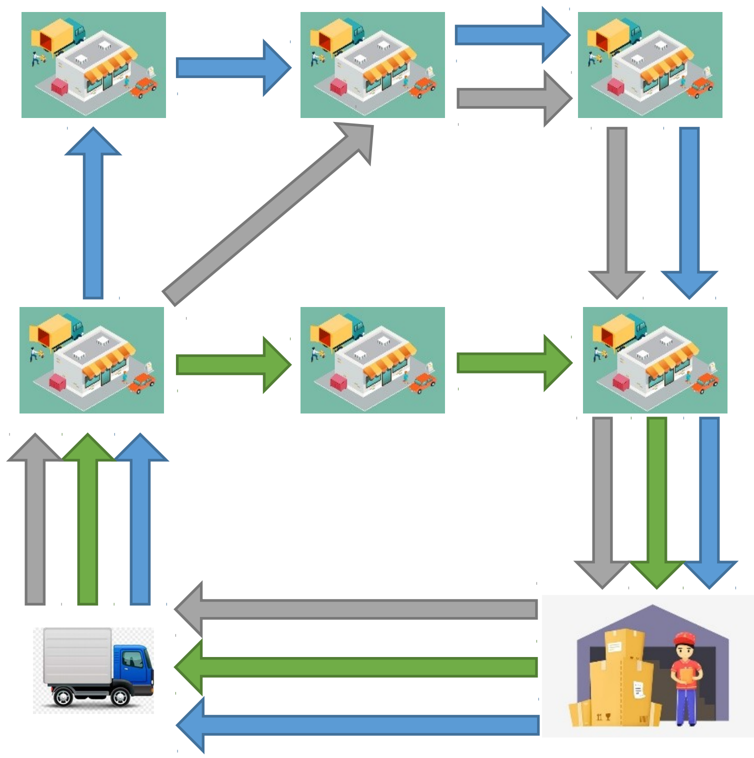


Fig. 1: Routes between stores 1,2,3,4,5,6 and WH.

The routes express the different existing paths to visit the stores according to the existing roads. In the illustrative example of Fig.1 there are three different routes: blue, green and gray. For example, the blue route links stores 1, 2, 3, 4, and 6 to form a loop that allows the truck to go back and forth to the storage. By following this route, the truck can visit all the stores or some of them, even several trucks can use that route visiting different stores, depending on the amounts to be delivered.

The input data needed is the following:

- Store locations *(i)*
- Existing and future locations of WH *(r)* and XD *(k)*.
- The store´s demand at time t *($demand_{it}$)*.
- The mix of products to be delivered to a store.
- The volume of each product in m^3.
- The delivery cost per truck size in $/km (*$costkm_c$*).
- Labor *($laborCost0_r$, $laborincr_j$)* and stock cost *($stock0_r$, stockcost)*.
- Operation cost of WH *($opercost0_r$, $operincr_j$)* and XDs (*$costxd_k$*).
- Truck capacities *(cap_c)*.
- WH´s capacities *($capWH0_r$, $capWHJ_j$)*.
- Delivery routes.
- Distances among WH and stores *($dist_{r,i}$)*.
- Investment costs for capacity expansion in WH *($invcost_j$)*.
- Rental cost of third-party warehouses *(costrent)*.

The decision variables are:
Investment decisions:

- The time and cost of installing a WH ($Invcost_{r,t}$)*or* a XD $TripAm_{c,k,i,t}$.
- The time and cost of capacity expansion in a WH *($CapWH_{r,t}$, $CapWHupJ_{r,t}$)*
- The time and cost of closing WH.

Operation decisions:

- Number of trips from WHs *($TripAm_{c,r,i,t}$)*, XDs *($TripAm_{c,k,i,t}$)* to the stores and its cost ($CostRI_{r,I,t}$, $CostKI_{k,I,t}$)
- Merchandise flows from WHs, XDs to the stores and the delivery route per time period *($FRI_{r,i,t}$, $FRK_{r,k,t}$)*.
- The amount of extra storage space rented (m^3) and the cost per year *($Rent_{r,t}$)*.

At the end of the manuscript the reader will be able to find the nomenclature section where the sets, parameters and variables used in the model are detailed.

3. PROPOSED MODEL

The proposed model corresponds to a multiperiod linear disjunctive one, formulated as a Generalized Disjunctive Programming (GDP) (Raman and Grossmann, 1994). The objective function is to minimize the investment and operation cost of the store´s replenishment and its structure over a total period of *t* months.

Discrete decisions involve the operation of new WH or XD facilities. Existing warehouses (WH) can be expanded or closed. Existing XD can continue its operation or can be closed. Since terms of disjunctions are exclusive or, only one term at time t can be true, meaning that in a specific location can be operating a WH or a XD or nothing. These decisions are posed by Eq. [1].

$$¿\left[XWH_{r,t}\ ¿ \right] \left[CapWH_{r,t} = capWH0_r\ ¿ \right] \left[OperCost_{r,t} = operCost0_r\ ¿ \right] \left[LaborCost_{r,t} = laborCost0_r\ ¿ \right] ¿\ ¿$$

The first term of disjunction [1] is handled by Boolean variable $XWH_{r,t}$, when true a warehouse can be installed or expanded at time t and operation ($opercost_{r,t}$) and labor cost ($laborcost_{r,t}$) apply. The storage capacity ($CapWH_{r,t}$) can be the one at the beginning of period t ($capWH0_r$) or can be increased by modules of discrete size; when Boolean variable $Z_{j,r,t}$ is true, the warehouse capacity is increased $capWHJ_j$ size and the labor and operation cost are raised $labincr_{r,j,t}$ and $operincr_{r,j,t}$, respectively. The inner disjunction handled by variable $yri_{r,i,i'}$ expresses that if warehouse r is used at time t, and exists a delivery route between r and store i, then there is a flow of products from r to i at time t ($FRI_{r,i,t}$) which is the summation of the amount of trips ($TripAm_{c,r,i,t}$) multiplied by the capacity of the trucks employed (cap_c).

When $XD_{k,t}$ variable is true a cross docking operates at location k, a fixed cost ($costxXD_k$) per volume handled is applied. Inner disjunction is related with the merchandise flow between WH and XD and between XD and shop i. If there is a route between k and i then $yki_{k,i}$ is true, then the merchandise flow $FKI_{k,i}$ is calculated by the summation of the amount of trips ($TripAm_{c,k,i,t}$) multiplied by the capacity of the trucks (cap_c). The same for r and k, in this case Boolean variable $ykr_{r,k}$ must be true. Both flows must be equal according to Eq. [11].

The third term of disjunction [1] with the negation of variables ¬XWHr,t and ¬XDk,t expresses that no WH and XD can be located at r and k respectively and therefore the costs involved are zero.

Warehouse capacity at time $t+1$ is equal to its capacity at time t plus the increase at time t (Eq. [2]), t_N is the last year of the time horizon.

$$CapWH_{r,t+1} = CapWH_{r,t} + CapWHupJ_{r,t} \quad \forall r \in R,\ 1 \le t \le t_{N-1} \tag{2}$$

Eq. [3] and Eq. [4] are like Eq. [2] to express the increase in operation and labor costs because of the capacity expansion for WH r at time t.

$$OperCost_{r,t+1} = OperCost_{r,t} + OperupJ_{r,t} \quad \forall r \in R,\ 1 \le t \le t_{N-1} \tag{3}$$

$$LaborCost_{r,t+1} = LaborCost_{r,t} + LaborupJ_{r,t} \quad \forall r \in R,\ 1 \le t \le t_{N-1} \tag{4}$$

The cost of an open warehouse r at time t corresponds to the labor ($LaborCost_{r,t}$) plus the operating cost ($OperCost_{r,t}$) as shown in Eq. 5.

$$CostR_{r,t} = LaborCost_{r,t} + OperCost_{r,t} \quad \forall r \in R,\ \forall t \tag{5}$$

The cost of using XD k for store i at time t is equal to the operation cost of the XD ($CostXD_{k,t}$) times the merchandise flow between them Eq. [6].

$$CostKI_{k,i,t} = CostXD¿¿_{k,t}¿¿ * FKI_{k,i,t} \quad \forall k \in RK, \forall i \in KI, \forall t¿$$

[6]

Transportation cost (Eq. 7) between WH r and store i is equal to the amount of trips among them ($TripAm_{c,r,i,t}$) times de kilometer cost ($costkm_c$) of truck c, according to its capacity, multiplied by the distance between r and i ($dist_{r,i}$).

$$CostRI_{r,i,t} = \sum_{c} costkm_c * TripAm_{c,r,i,t} * dist_{r,i} \quad r \in R, \forall i \in RI, \forall t$$

[7]

The stock of WH r at time t is a percentage (perc) of his full capacity to allow the merchandise movement inside the facilities (Eq. [8]).

$$Stock_{r,t} = perc * CapWH_{r,t} \quad r \in R, \forall t$$

[8]

Eq. [9] shows the merchandise flow balance in the WH plus its stock, at time $t=t_1$. Initial stock at WH r $(stock0_r)$ plus the input flow $Flow_{r,t}$ is equal to the summation of output flows to shop i $(FRI_{r,i,t})$ plus the summation of output flows to XD k $(FRI_{r,k,t})$ plus the remaining stock at time t $(Stock_{r,t})$ and extra space rented to third-party at time t $(Rent_{r,t})$, for initial time t_1.

$$stock0_r + flow_{r,t} = Stock_{r,t} + Rent_{r,t} + \sum_{i \in RI} FRI_{r,i,t} + \sum_{k \in RK} FRK_{r,k,t} \quad \forall r \in R, t = t_1$$

[9]

Eq. [10] establishes that the stock at time $t-1$ of WH r $(Stock_{r,t-1})$ plus the input flow $Flow_{r,t}$ at time t is equal to the summation of output flows to stores i $(FRI_{r,i,t})$ plus the summation of output flows to XD k $(FRI_{r,k,t})$ plus the remaining stock at time t $(Stock_{r,t})$ and extra space rented to third-party at time t $(Rent_{r,t})$, $\forall t \geq t_2$.

$$Stock_{r,t-1} + flow_{r,t} = Stock_{r,t} + Rent_{r,t} + \sum_{i \in RI} FRI_{r,i,t} + \sum_{k \in RK} FRK_{r,k,t} \quad \forall r \in R, \forall t \geq t_2$$

[10]

For XD k, the summation of input flow $(FRK_{r,k,t})$ is equal to the summation of the output $(FRI_{r,i,t})$ Eq. [11].

$$\sum_{r} FRK_{r,k,t} = \sum_{i} FKI_{k,i,t} \quad \forall k \in K, \forall t$$

[11]

The summation of flows to store i from WH r and XD k should cover the demand of store i at time t if a route exists from WH r to XD k, and from k to store i (Eq. [12]). The demand ($demand_{i,t}$) can be covered from deposit r if there exists a route that passes through shop i or ends at that point. The variable y of Eq. 12 represents that decision in the model.

$$\sum_{r \in RI} ¿\, i' \in RXX ¿ + \sum_{k \in KI} yki_{k,i}) \quad \forall i, \forall t ¿$$

[12]

A transformation is needed for the hierarchical decisions of disjunction [1] such that the problem is posed as a GDP formulation. Inner disjunctions must be taken out as a single one and several logical constraints must be added to relate the variables handling the disjunction´s terms (Vecchietti and Grossmann, 2000). These transformations are presented in the following paragraphs.

Disjunction [13] corresponds to the decision of installing or not a WH and a XD. Cost constraints apply for each case.

$$¿\left[XWH_{r,t} ¿\right]\left[CapWH_{r,t} = CapWH0_r ¿\right]\left[OperCost_{r,t} = operCost0_r ¿\right]\left[LaborCost_{r,t} = laborCost0_r ¿\right] ¿ ¿$$

$$¿$$

Disjunction [14] represents the capacity expansion of a WH and also the rise in labor and operation cost. Since disjunction is now independent of the hierarchical decision, a negative term ($\neg Z_{r,j,t}$)is added for the case where an existing WH is not expanded or is closed; in this case the increase in capacity and costs are zero.

$$¿ \quad ¿\left[Z_{r,j,t} ¿\right]\left[CapWHupJ_{r,t} = capWHJ_j ¿\right]\left[LaborupJ_{r,t} = labincr_{r,j,t} ¿\right] ¿ ¿$$

$$¿$$

[14]

Like Eq. [14], disjunction [15] is extracted and a negative term ($\neg y_{r,j,i'}$) is added to consider the case where an existing WH is closed and therefore the merchandise flow between WH and stores is zero.

$$¿ \quad ¿\left[yri_{r,i,i'} ¿\right] ¿ ¿$$

$$¿$$

[15]

Disjunctions [16] and [17] are transformed likewise than [14] and [15] for the case where a cross docking is not operated at that location. For this case, input and output flows from XD k are zero.

$$\text{¿} \quad \text{¿}\left[yki_{k,i\,\text{¿}} \text{¿} \right] \text{¿} \; \text{¿}$$

[16]

$$\text{¿} \quad \text{¿}\left[yrk_{r,k\,\text{¿}} \text{¿} \right] \text{¿} \; \text{¿}$$

[17]

According to Eq. [18] a WH or a XD can operate at the same location, or none of them.

$$XWH_{r,t} + XD_{k,t} \leq 1 \quad \forall r \in R, \forall k \in RK, \forall t$$

[18]

Eq. [19] represents whether a WH can have a capacity expansion j over the time horizon.

$$\sum_j Z_{r,j,t} \leq XWH_{r,t} \quad \forall r \in R, \forall t$$

[19]

Eq. [20] establishes that a delivery route can exist (or not) between WH r and store i over the time horizon.

$$\sum_{i'} yri_{r,i,i'} \leq XWH_{r,t} \quad \forall r \in R, \forall i \in RI, \forall t$$

[20]

Eq. [21] expresses that a delivery route can exist (or not) between XD k and store i over the time horizon. Similar for Eq. [22] for paths between WH r and XD k.

$$\sum_i yki_{k,i} \leq XD_{k,t} \quad \forall k \in K, \forall t$$

[21]

$$\sum_r yrk_{r,k} \leq XD_{k,t} \quad \forall k \in K, \forall t$$

[22]

Eq. [23] states that the route must have a store at the end of it such that its distance is greater than the previous points.

$$dist_{r,i} * yri_{r,i,i'} \geq dist_{r,i'} * yri_{r,i',i} \quad \forall (r,i,i') \in Rxx, \forall (r,i,i') \in RI$$

[23]

Eq. [24] establishes that every store must be visited a minimum number of times *(minv)* per period.

$$\sum_{c,r \in RI} TripAm_{c,r,i,t} + \sum_{c,k \in KI} TripAm_{c,k,i,t} \geq minv \quad \forall i, \forall t$$

[24]

OBJECTIVE FUNCTION

The objective function is the cost minimization of the whole logistic operation (Eq.25), which includes the total operation and investment costs. Since the model is a multiperiod one the cost is actualized with an annual interest rate *(Tax)*.

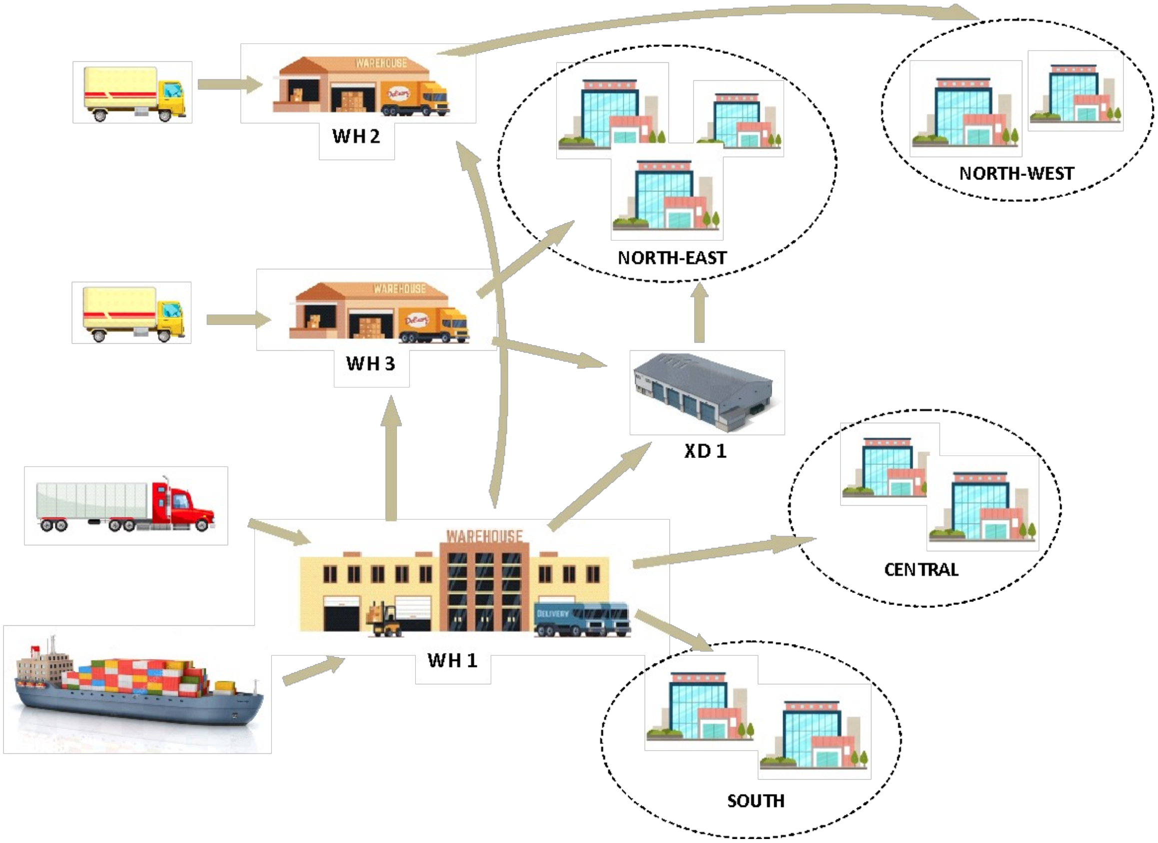
$$TotalCost = \sum_{r,t}^{❑} ❑ \frac{InvCost_{r,t}}{¿¿}$$

Eq. 26 determines the annual operational cost that includes operating, labor, stock, transportation, and rental.

$$Cost_t^{❑} = \sum_{r,t \in year}^{❑} ❑ \left(CostR_{r,t}^{❑} + stockcost . Stock_{r,t} \right) + \sum_{r,i,t \in year}^{❑} ❑ CostRI_{r,i,t} + \sum_{k,i,t \in year}^{❑} ❑ CostKI_{k,i,t} + \sum_{r,t \in year}^{❑} ❑ costrent$$

4. CASE STUDY

The case study corresponds to a real case of an electronic and appliance sale company having 121 shops located around a large geographical region. The structure to supply the stores (Fig. 2) is the following: three warehouses, a big central one (WH1), located near a port where many of the products are imported, and two more (WH2 and WH3), smaller in size, located in specific regions which provide goods to the stores close to them. In Addition, a cross docking (XD1) managed by a third-party is used to satisfy another store set. It is important to note that, in this structure, 95% of the input flow of merchandise is collected in WH1, only 5% is sent to the smaller WHs. This



means that WH1 delivers merchandise to the other WHs, XD1 and some sale stores.

Fig. 2: Current structure of warehouses, cross docking and stores of the Case Study

The objective of the study is to analyze the delivery cost of replenishing merchandise from WHs to the stores over a period of 10 years. The company seeks to analyze the possibility of closing WH2 and WH3 or replacing them by XDs operated by third-parties, or the option to increase their capacities, including WH1. Stores are located between a few kilometers to several hundred kilometers from WH1 which centralizes and distributes the merchandise.

The stores are clustered in four regions according to their locations: North-East, North-West, Central and South. With these definitions, North-West stores are supplied by WH2, North-East´s by WH3 and XD1, while WH1 delivers products to Central and South retailers and to XD1, WH2 and WH3. Between two points of the structure there exists several paths, but only one of them can be chosen according to the delivery to be made. The cluster's definition reduces the combinatorial path selection. Besides, this assumption is supported by the highways/routes connecting different places. The model also decides the truck type to perform the delivery, where several sizes are available according to the route and the amount to deliver.

The demand projection over the years is proposed by the company taking into account the historical sale values of previous years. The analysis of those values shows that there are seasonal demands according to holidays, special dates (Christmas time for example) and sale promotions during specific times of a calendar year (Black Friday, Cyber Monday). This also has an impact on the delivery and the capacity requirements of the WHs, sometimes extra storage space is rented to third parties to keep the necessary stock. Fig. 3 shows the volumetric demand corresponding to a calendar year from august 2018 until July 2019. Merchandise purchases are made in advance so that the need for extra space to maintain stock can be programmed beforehand. This situation is contemplated in the model.

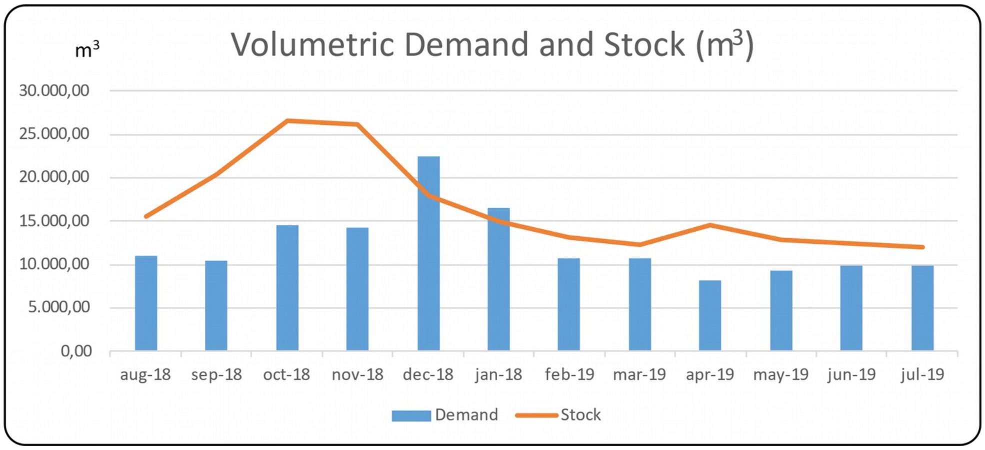


Fig. 3: Volumetric demand and stock data in a operation year (m^3)

5. RESULTS

To solve the model, the disjunction´s terms are relaxed by means of Big-M formulations (Vecchietti et al., 2003). The transformed model is a mixed-integer linear programming problem (MILP) which was posed in the GAMS system (Brooke et al., 1998) and solved with CPLEX 12.6.3 in a personal computer with an Intel i7 processor with 8 GB of RAM. The model consists of 472522 equations, 429885 variables, and 82880 binary variables (0-1). It took 497 CPU seconds to reach the solution.

To analyze the quality of the solution, a comparison is made between the optimal results obtained with the model and a simulation of the current situation of the company.

Fig. 4 shows the structure of the delivery logistic proposed by the model, where it can be seen that WH2 is replaced by a third-party cross docking terminal and WH3 is closed. The distribution made by WH3 is absorbed by XD1 and WH1. The purchased merchandise is stored only in WH1 and then distributed to the rest of the facilities. The solution proposes two capacity extensions for WH1. The expansions are formulated in a discrete manner in modules of 6.000, 12.000 or 18.000 m3. The model decided two expansions of the minimum module (6.000 m3 each one), one at the beginning of the horizon time (2019) and the other, five years later (2024), with a total cost of $ 1.280.000,00 ($640.000 each one). The investment cost in expansions is low compared with transportation or warehouse operation costs. The first expansion is decided to absorb the volume of the closed warehouses, the next one to diminish the storage capacity rented to third-parties.

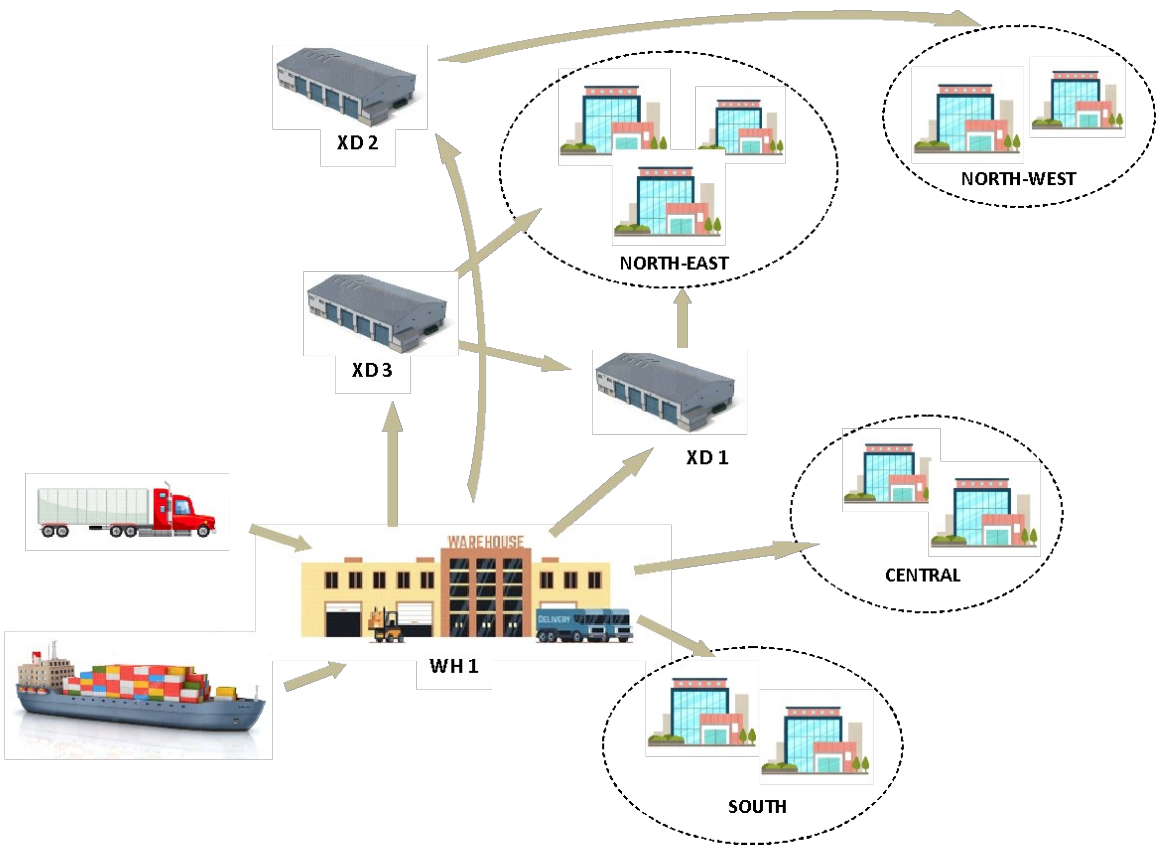


Fig. 4: Structure proposed by the model optimal solution of the Case Study

A summary of the results is presented in Fig. 5. The optimal solution obtained with the model is compared with the current situation, which was simulated in order to compare the values. The figure shows different components of the total cost to facilitate the results analysis. The first bar in Fig.3 corresponds to WHs costs, which includes labor, the operating costs such as electricity, water, cleaning, security, facilities maintenance. The closure of WH2 and WH3 allows an important saving related to operational cost. The next item is the rental of third-parties deposits on a seasonal basis, at peak times in the stock. In the optimal solution that cost is increased significantly with respect to the current situation, this is mainly due to the decrease in storage capacity because of the closure of WH2 and WH3.

Transportation is the third item to compare; this item corresponds to the merchandise delivery from the warehouse to the shops, a significant increase because the contract with the new XD includes the transportation cost which is higher than using independent companies. Due to the closure of two warehouses, all the merchandise purchased is stored in WH1 and then distributed from it, also new points are supplied from WH1.

The fourth item is cross docking cost which is based on the volume handled and includes the physical use of space. This is a bit higher with respect to the simulation; the volume handled by the new facility (XD2) is not important and its cost is lower than XD1 because of its geographical location. The stock cost is similar to the current situation since its volume does not change significantly.

Finally, the figure compares the total cost showing a lower value than with respect to the simulation of the current situation.

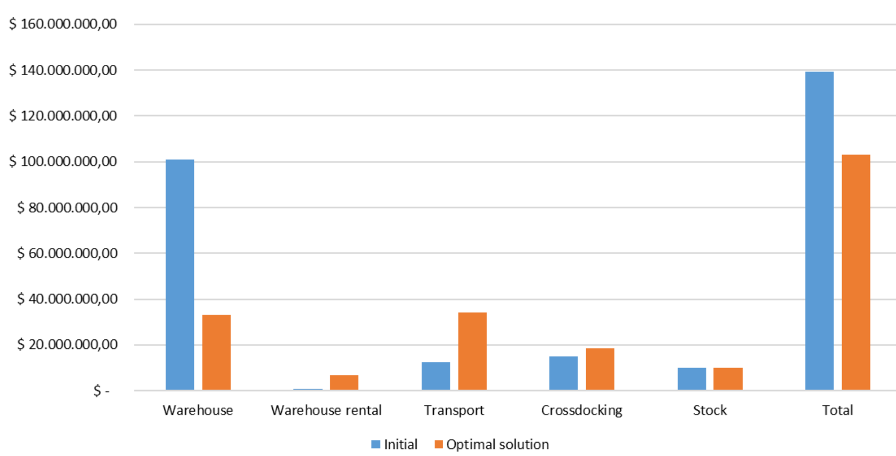


Fig. 5: Cost comparison – Model Optimal Solution vs. current situation.

The cost values presented in Fig. 5 are broken down in Table 1 and Table 2 showing the evolution in dollars. The last row shows the estimated cost per cubic meter, which decreases over the years because a greater volume is moved having almost the same structure. It also shows a lower value of the optimal solution of the model.

In table 1, a gradual growth of the annual cost can be observed, the same occurs for all the items considered except in the costs of deposits which remain at the same value, since no capacity extensions are made, neither in labor nor services. The demand grows annually according to the estimation provided by the company, the volume of merchandise grows, making the relative cost per cubic meter decreasing over the time horizon, in both scenarios. Also because of the increasing demand, the volume rented to third-party warehouses increases over the years.

Similar behavior showing in Table 1 occurs for the optimal solution scenario in Table 2. While transportation and cross docking cost increases with respect to the current situation, the operational cost of warehouses is much lower compensating the rises. Stock cost remains similar. On the other hand, the cost of warehouses gradually increases because of the hiring of personnel due to the growth of the volume handled at WH1.

Tabla 1: Current situation simulation cost (u$s)

	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Total	$ 13.453.463,64	$ 13.534.793,55	$ 13.620.887,50	$ 13.714.107,42	$ 13.825.337,53	$ 13.961.054,02	$ 14.106.126,50	$ 14.250.307,27	$ 14.400.842,32	$ 14.558.228,59
Transport	$ 1.159.920,06	$ 1.175.517,35	$ 1.193.906,70	$ 1.217.390,87	$ 1.242.823,47	$ 1.265.257,14	$ 1.293.648,31	$ 1.317.647,35	$ 1.344.395,20	$ 1.374.280,66
Crossdocking	$ 1.311.014,22	$ 1.350.344,65	$ 1.390.854,99	$ 1.432.580,64	$ 1.475.558,06	$ 1.519.824,80	$ 1.565.419,54	$ 1.612.382,13	$ 1.660.753,59	$ 1.710.576,20
Warehouse rental	$ 0,00	$ 0,00	$ 0,00	$ 0,00	$ 13.969,70	$ 53.269,87	$ 93.749,05	$ 135.442,60	$ 178.386,96	$ 222.619,65
Stock	$ 880.073,23	$ 906.475,43	$ 933.669,69	$ 961.679,78	$ 990.530,18	$ 1.020.246,08	$ 1.050.853,47	$ 1.082.379,07	$ 1.114.850,44	$ 1.148.295,95
Warehouse	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12	$ 10.102.456,12
US$ for m3	$ 87,14	$ 85,11	$ 83,16	$ 81,29	$ 79,56	$ 78,00	$ 76,52	$ 75,05	$ 73,63	$ 72,27

Tabla 2: Optimal solution costs (u$s).

	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
Total	$ 9.302.596,23	$ 9.417.284,43	$ 9.652.876,00	$ 9.888.201,91	$ 10.121.969,73	$ 10.422.509,42	$ 10.643.694,73	$ 10.887.604,32	$ 11.174.570,72	$ 11.444.423,42
Transport	$ 3.044.130,62	$ 3.116.062,74	$ 3.206.117,56	$ 3.286.578,51	$ 3.357.767,10	$ 3.447.470,87	$ 3.532.563,96	$ 3.614.882,36	$ 3.735.409,84	$ 3.833.723,32
Crossdocking	$ 1.620.788,54	$ 1.669.412,19	$ 1.719.494,56	$ 1.771.079,39	$ 1.824.211,78	$ 1.878.938,13	$ 1.935.306,27	$ 1.993.365,46	$ 2.053.166,42	$ 2.114.761,42
Warehouse rental	$ 621.530,77	$ 589.260,99	$ 657.521,11	$ 732.791,14	$ 813.387,60	$ 866.420,46	$ 548.733,20	$ 620.739,60	$ 694.906,18	$ 771.404,89
Stock	$ 880.073,23	$ 906.475,43	$ 933.669,69	$ 961.679,78	$ 990.530,18	$ 1.020.246,08	$ 1.050.853,47	$ 1.082.379,07	$ 1.114.850,44	$ 1.148.295,95
Warehouse	$ 3.136.073,08	$ 3.136.073,08	$ 3.136.073,08	$ 3.136.073,08	$ 3.136.073,08	$ 3.209.433,87	$ 3.576.237,84	$ 3.576.237,84	$ 3.576.237,84	$ 3.576.237,84
US$ for m3	$ 60,25	$ 59,22	$ 58,93	$ 58,61	$ 58,25	$ 58,23	$ 57,74	$ 57,34	$ 57,14	$ 56,81

6. CONCLUSIONS

This article presents a disjunctive programming model to decide the optimal facilities and delivery routes to replenish merchandise to the shops of a retail company. Discrete decisions are modeled using disjunctions to decide if a warehouse will be installed, closed, or increase the capacity; or a new cross docking is necessary. The model corresponds to a strategic one, applied over a horizon time of ten years. Several costs are considered in the objective function: investment, operational, transportation, stock, and rental costs. To analyze the capabilities of the proposal, the model is applied to a case study corresponding to an electronic and appliances retail company having stores distributed over a wide geographical region.

The proposed model is a valuable analysis tool to study delivery structures for several businesses, not only retail. Several scenarios can be analyzed by varying the parameters and conditions of the delivery structure. Business managers can use it to make decisions to improve competitiveness by diminishing logistic operation cost for companies where it entails an important portion of the total operation. This model provides a systemic way to study the distribution problem of a company and, with some adaptations, it could be transformed into an operational model to supply merchandise over stores located in different locations.

The results of the case study show that a saving of around 30% of the total distribution cost is possible to obtain by closing one WH and replacing the other by a third-party cross docking. Delivery routes are automatically selected by the model because of the new delivery structure. Savings are mainly obtained in labor and operational costs of warehouses; stock costs remain similar while transportation costs increase because of the use of cross dockings and the selection of delivery routes.

NOMENCLATURE

SETS

c: trucks
i: stores/shops
j: interval of capacity increase
k: cross dockings locations
r: warehouses locations
t: time periods

PARAMETERS

cap_c: *capacity of truck c*
$capWH0_r$: *initial capacity of warehouse r*
$capWHJ_j$: *capacity increase of interval j*
$costkm_c$: *cost per kilometer of truck c*
costrent: rental average cost of third-party warehouse space
$costxd_k$: *cross-docking cost per volume handled of XD k*
$demand_{i,t}$: *merchandise volume (in* m^3*) to deliver to store i at time t*
$dist_{r,i}$: *distance in km between warehouse r and shop i*
$flow_{r,t}$: *merchandise purchased and received at warehouse r at time t* (m^3)
$invcost_j$: *investment cost of interval j to increase capacity to a warehouse ($)*
$laborCost0_r$: *initial labor cost of warehouse r ($)*
$laborincr_r$: *labor cost increase of interval j ($)*
minv: minimum number of trips to deliver goods to store i
$operCost0_r$: *initial operation cost of warehouse r ($)*
$operincr_j$: *operation cost increase of interval j ($)*
perc: maximum percentage of full warehouse capacity to store goods
$stock0_r$: *initial stock of warehouse r* (m^3)
stockcost: average financial cost ($\$/m^3$)
tax: interest rate

VARIABLES

Boolean

$XWH_{r,t}$:when true wearhouse r is operating at time t otherwise not
$XD_{k,t}$: when true cross-docking k is operating at time t instead of warehouse r otherwise not
$Z_{r,j,t}$: when true a capacity of size j is done for warehouse r at time t otherwise not
$yri_{r,i}$: when true a merchandise delivery is performed from warehouse r to store i at time t otherwise not
$yki_{k,i}$: when true a merchandise delivery is performed from XD k to store i at time t otherwise not
$yrk_{r,k}$: when true a merchandise delivery is performed from warehouse r to XD k at time t otherwise not

Continuous

$CapWH_{r,t:}$: capacity in m^3 of warehouse r at time t
$OperCost_{r,t}$: operational cost of warehouse r at time t
$LaborCost_{r,t}$: labor cost of warehouse r at time t
$CapWHupJ_{r,t}$: capacity increase of warehouse r at time t
$OperupJ_{r,t}$: operation cost increase of warehouse r at time t
$LaborupJ_{r,t}$: labor cost increase of warehouse r at time t
$CostR_{r,t}$: operational cost of warehouse r at time t (includes labor and operation)
$CostKI_{k,I,t}$: transportation cost from cross-docking k to shop i at time t
$CostRI_{r,I,t}$: transportation cost from warehouse r to shop i at time t
$CostXD_k$: cross-docking cost k per volume handled (\$/m^3)
$FKI_{k,I,t}$: volume delivered (m^3) from XD k to shop i at time t
$FRI_{r,i,t}$: volume delivered (m^3) from warehouse R to shop i at time t
$FRK_{r,k,t}$: volume delivered (m^3) from warehouse R to XD k at time t
$Rent_{r,t}$: space rented (m^3) to a third-party warehouse
$Stock_{r,t}$: stock (m^3) of warehouse r at time t

Integer

$TripAm_{c,k,i,t}$: number of trips of truck c from XD k to shop at time t
$TripAm_{c,r,i,t}$: number of trips of truck c from warehouse r to shop i at time t

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