Symbolic Time Series and Causality Detection: an Uneasy Alliance

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Jornadas Interdisciplinarias en Sistemas Complejos 2021

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Motivation

- Some economic and social time series have a "noisy" structure.
- Symbolic time series analysis is a useful technique to reduce the dimension of the realization space of the series.
- We wonder if causality relationship is a property that remains invariable after the data is transformed to a symbolic expression.
- Exercises: simulation and observational data.

Symbolic Time Series Analysis in Economics

Symbolic Aggregate Approximation (SAX) (Lin et. al., 2007)

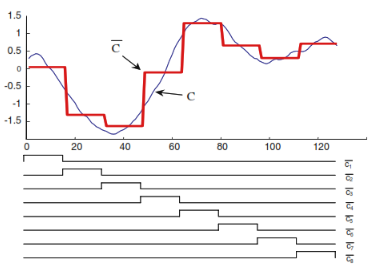
- SAX allows a time series of arbitrary length n to be reduced to a string of arbitrary length $w < n$
- The alphabet size is also an arbitrary integer $a > 2$.
- Steps:
  1. Data transformation for dimensionality reduction: Piecewise Aggregate Approximation (PPA)
  2. Symbolize the PPA representation into a discrete string

SAX: dimensionality reduction via PAA

- A time series C of length n can be represented by a vector $\bar{C} = \bar{c}_1, \ldots, \bar{c}_w$. in a w-dimensional space.

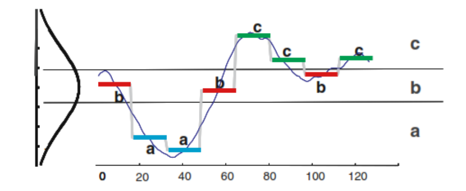
The ith element of $\bar{C}$ is calculated by

$$\bar{c}_i = \frac{w}{n} \sum_{j=\frac{n}{w}(i-1)+1}^{\frac{n}{w}i} c_j$$



SAX: discretization

1. Consider a normalization of the time series
2. Identify the *breakpoints*: a sorted list of numbers $B = \beta_1, \ldots, \beta_{|d|-1}$ such that the area under a $N(0,1)$ Gaussian curve from β_i to β_{i+1} equals $\frac{1}{|d|}$.
3. Define the alphabet of symbols. Let α_i denote the ith element of the alphabet. i.e. $\mid d \mid = 3$, $d = \{\alpha_1 = \mathbf{a}, \alpha_2 = \mathbf{b}, \alpha_3 = \mathbf{c}\}$.
4. Map PAA approximation $\bar{C}$ to a word $\hat{C} = \hat{c}_1, \ldots, \hat{c}_w$ as follows: $\hat{c}_i = \alpha_j$, iif $\beta_{j-1} \leq \bar{c}_i < \beta_j$.



Lin et. al. 2007

Markov-switching Models (MSwM)

Markov-switching models: A two state Markov chain

- MSwM allows to characterize how a non-stationary series transitions between different regimes, drawing the probability distribution of the switches between those regimes.
- Under the MSwM, a regime is the equivalent of a symbol in STSA.

Markov-switching models: definitions

Let be a system with a finite number of states $1, \ldots, N$, such that any period $t \in \mathbb{N}$ the distribution of possible instations of the state variable s_t satisfies the following condition:

$$P\left\{s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots\right\} = P\left\{s_t = j | s_{t-1} = i\right\} = p_{ij}$$

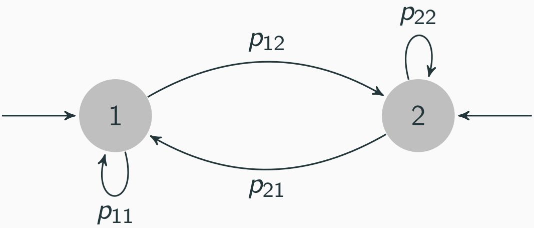
with $p_{i1} + p_{i2} + \cdots + p_{iN} = 1$.
Each p_{ij} represents the probability of the transition from state i to state j.

Markov-switching models: transition matrix and steady state of the system.

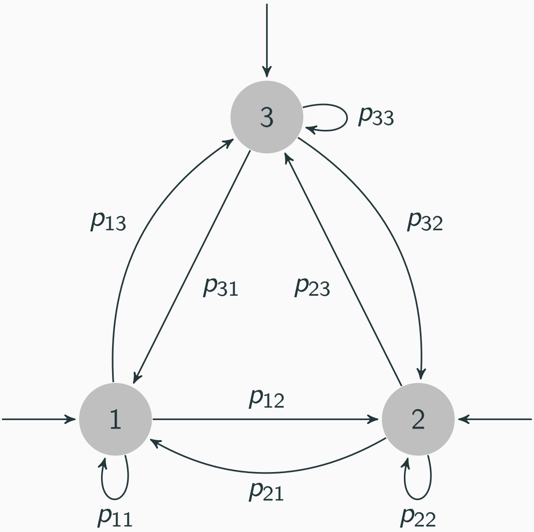
$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}$$

- P be the transition matrix
- the *steady state* of the system is understood as an N-components vector $\pi = (\pi_1, \ldots, \pi_N)$ such that each π_i is the long-term probability of finding the system at state i.
- $\sum_i^N \pi_i = 1$, then π satisfies $P\pi = \pi$.
- If $\lambda_1 = 1$ is the first eigenvalue of P, as indicating that π is its associated eigenvector.

Markov-switching models: A two state Markov chain



Markov-switching models: A three state Markov chain



Markov-switching models: time series system

- Time series: $\{y_t\}_{t\geq 0}$, $y_t \in Y$. F_t: distributions over Y at t.
- Transitions between regimes:

For $1, \ldots, N$ regimes,

$$y_t - \mu_{s_t^*} = \phi\left(y_{t-1} - \mu_{s_{t-1}^*}\right) + \varepsilon_s$$

where $\mu_{s_t^*} \in Y$ corresponds to the state $s_t^* \in \{1, \ldots, N\}$.

- If $s_t^* = j$ and $s_{t-1}^* = i$, at $t-1$, μ_i is followed in t by μ_j, with $\mu_i \neq \mu_j$.
- The transition from $\mu_{s_{t-1}^*}$ to $\mu_{s_t^*}$, corresponding to transition from state j to state i has probability p_{ij}.
- ϕ is a function that embodies the combined action of P and, for each state i and period t, the conditional distribution $F_t(y|i)$.

Causality analysis with time series data

Transfer Entropy

- Rényi entropy: $H_q = \frac{1}{1-q} \log \left(\sum_{i=1}^{n} p_i^q \right)$

Transfer entropy:

$TE(X \to Y) = H(Y_t \mid Y_{t-1:t-p}) - H(Y_t \mid Y_{t-1:t-p}, X_{t-1:t-p})$

Test: $TE(X \to Y) = 0$

Granger causality test

Causality: $X \to Y$ and $Y \nrightarrow X$

VAR model:

- $Y_t = \delta + \sum_{j=1}^{p} \theta_{11,j} Y_{t-j} + \sum_{j=1}^{p} \theta_{12,j} X_{t-j} + u_{Yt}$
- $X_t = \eta + \sum_{j=1}^{p} \theta_{21,j} Y_{t-j} + \sum_{j=1}^{p} \theta_{22,j} X_{t-j} + u_{Xt}$

X "Granger-cause" Y if:

- $E(Y_t \mid Y_{t-1}, \ldots, Y_{t-p}, X_{t-1}, \ldots, X_{t-p}) \neq E(Y_t \mid Y_{t-1}, \ldots, Y_{t-p})$, and
- $E(X_t \mid X_{t-1}, \ldots, X_{t-p}, Y_{t-1}, \ldots, Y_{t-p}) = E(X_t \mid X_{t-1}, \ldots, X_{t-p})$

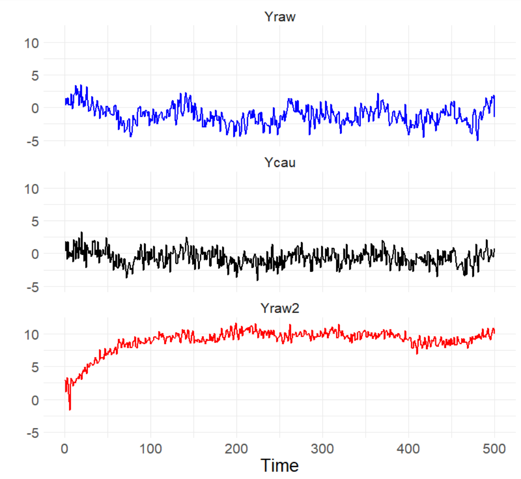
Test: $H_0 : \boldsymbol{\theta_{12,j}} = 0$ and $H_0 : \boldsymbol{\theta_{21,j}} = 0$

Exercises with data

Exercise 1: simulated time series

Raw time series: $y_t^R = \delta + \sum_{i=1}^{6} \theta_i y_{t-p}^R + u_t$

Caused time series: $y_t^C = \gamma + \phi_i y_{t-1}^R + v_t$

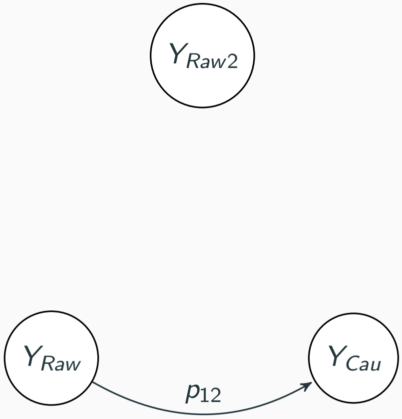


Exercise 1: causality between Yraw, Ycau and Yraw2

Without Dictionary (p-values)

	Caused					
	Yraw		Ycau		Yraw2	
Cause	TE (q=0.9)	Granger	TE (q=0.9)	Granger	TE (q=0.9)	Granger
Yraw			**0.04**	**2.20E-16**	0.9	0.1441
Ycau	0.3167	0.6127				
Yraw2	0.6767	0.9765				

Causal graph: raw series



Exercise 1: causality with transformed Yraw and Ycau

Yraw ⟶ Ycau? (pvalues)

Yraw Ycau	TE (q=0.9)	Granger
SAX g D2	0.0533	**0.0003148**
SAX g D3	0.0533	**0.0003148**
SAX g D4	0.0533	**0.0003148**
SAX q D2	**0.0133**	**8.84E-05**
SAX q D3	0.5	**6.86E-05**
SAX q D4	0.5467	**6.86E-05**
Markov D2	**0**	0.1
Markov D3	0.36	0.09582
Markov D4	**0**	0.7512

Ycau ⟶ Yraw? (pvalues)

Ycau Yraw	TE (q=0.9)	Granger
SAX g D2	**0.02**	**0.0002007**
SAX g D3	**0.01**	**0.0002007**
SAX g D4	**0.0067**	**0.0002007**
SAX q D2	**0.0133**	**6.89E-05**
SAX q D3	0.1267	**4.48E-05**
SAX q D4	0.1567	**4.48E-05**
Markov D2	**0**	**6.86E-05**
Markov D3	0.8433	0.5768
Markov D4	**0**	0.5705

Exercise 1: causality with transformed Yraw and Yraw2

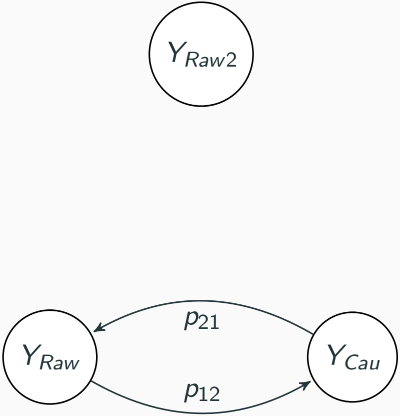
Yraw $\longrightarrow$ Yraw2? (pvalues)

Yraw Yraw2	TE (q=0.9)	Granger
SAX g D2	0.2933	0.5903
SAX g D3	0.3267	0.5909
SAX g D4	0.3667	0.5908
SAX q D2	0.43	0.8221
SAX q D3	0.4367	0.8284
SAX q D4	0.42	0.8285
Markov D2	0.5	0.05735
Markov D3	0.79	0.4254
Markov D4	0	error

Yraw2 $\longrightarrow$ Yraw? (pvalues)

Yraw2 Yraw	TE (q=0.9)	Granger
SAX g D2	0.6567	0.06775
SAX g D3	0.67	0.06775
SAX g D4	0.6533	0.06775
SAX q D2	0.8467	0.3165
SAX q D3	0.76	0.1967
SAX q D4	0.7967	0.1967
Markov D2	1	0.9491
Markov D3	0.9433	0.4864
Markov D4	0	error

Causal graph: symbolic series

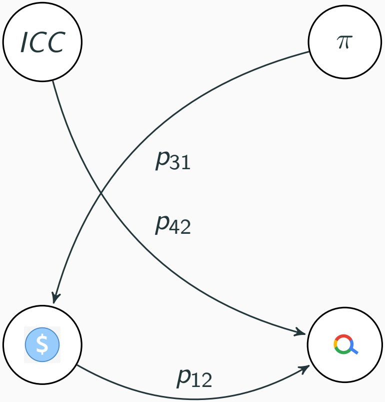


Exercise 2: Narratives Data

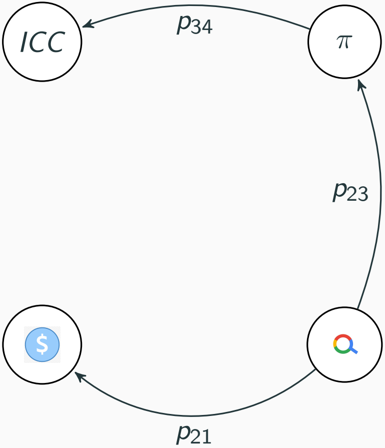
Narratives about dollar: variables

- Google searches: searches about "dólar blue" between 2004 and 2019.
- "Dólar blue": informal exchange rate between peso and dollar.
- ICC: consumer confidence index, measured by CIF-UTDT.
- Inflation rate (π): variation rate of the consumer price index.

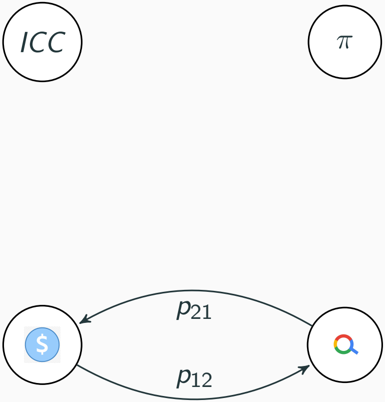
Without Alphabet (TE)



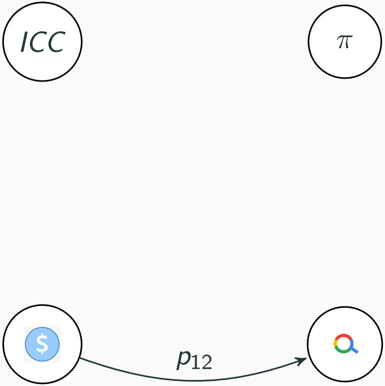
Without Alphabet (Granger)



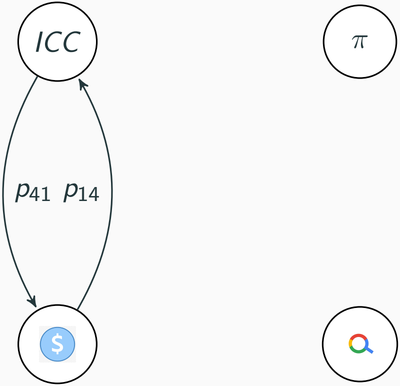
SAX Aphabet=2 (TE)



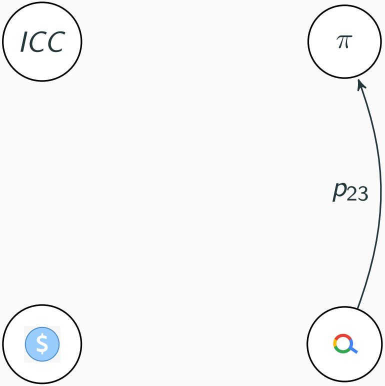
SAX Aphabet=2 (Granger)



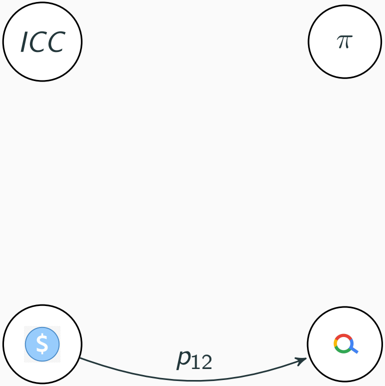
SAX Aphabet=3 (TE and Granger)



Markov Aphabet=2 (TE)



Markov Aphabet=2 (Granger)



Conclusions

- When the causal relationship is clear, causality test performs as expected with the untransformed data.
- With the data transformed to symbolic series, by the use of SAX or Markov switching model, the tests fail to detect the correct causal relation.
- With the observational data, where the causal relations are less neat, this problem is severe.
- Potential explanation: symbolic transformation distorts the relations between variables in a way that artificially generates causality which is mistakenly detected by the test.